Euler against Newtonian Gravity: "A Crude Hypothesis"?

ABSTRACT

This paper is a two-pronged response to George E. Smith's "Newton's numerator in 1685: A year of gestation", Studies in History and Philosophy of Modern Physics 68 (2019) 163-177. I offer this response from the perspective of Euler scholarship. First, I challenge Smith's claim that Euler dismisses gravity's proportionality to the mass of the attracting body. Rather than rejecting this proportionality from the numerator of Newton's law of gravity, I will show that Euler is opposed to Newton's appeals to the third law of motion to derive this term. Second, I proffer a critical assessment of Euler's derivation of all three proportionalities in Newton's law of gravity, a derivation that does not involve the third law, but the material properties of ether and contact action (i.e., fluid pressure). My analysis reveals that, while Euler is right to point out the lack of direct evidence for gravity being a force of interaction governed by the third law of motion, his alternative falls far short of its Newtonian rival on grounds of empirical support and fruitfulness to future research.

Introduction

Howard Stein argues in "From the Phenomena of Motion to the Forces of Nature: Hypothesis or Deduction?" that Newton's application of the third law of motion to attraction presupposes "the far-reaching hypothesis" that gravity is "a force of direct interaction between the heavy body and the central body toward which it has weight" (Stein 1999, pp. 219). As Stein correctly points out, the phenomena Newton cites show only that the heavy bodies are urged by a force in the direction of the corresponding central bodies. The phenomena do not show that the heavy bodies are acted on by the central bodies. It is therefore question-begging to apply the third law to attraction given the empirical evidence available at the time: the law holds only for pairs of bodies acting on each other. In a recent article published in this journal, George Smith brings our attention to a similar challenge posed by Euler against Newton. The challenge is articulated in a 1751 letter to Tobias Mayer:

> As for the cause of gravity which I have rejected, I consider the objection that the attraction must not necessarily be proportional to the masses, to be of no great importance, as it is *still not decided by any single phenomenon that the attractive forces of heavenly bodies are proportional to their masses*. On the contrary, Newton tried to determine the masses on this basis since there is no other way of specifying them. As soon as one now places the statement that the attractive forces are proportional to the masses (*which is founded on a crude hypothesis*) in doubt, this objection against my idea is eliminated.[1] [Smith's emphasis]

This is a response to a letter in which Mayer points out that Euler's proposed magnetic mechanism for universal gravity is inadequate to satisfy Newton's law. If gravity were produced by a magnetic pole, Mayer holds, the attraction would not remain proportional to the mass of the attracting body. Euler insists in response to Mayer that the supposed proportionality is not supported by any phenomenon. It is instead "founded on a crude hypothesis." Newton obtains the mass of the attracting body by assuming that the third law of motion holds for attraction. The consideration driving Euler's complaint therefore appears to be the same as that driving Stein's.[2] On Smith's view, the quoted letter indicates that Euler "had been even more dismissive [than Stein] of the claim that the mass of the attracting body belongs in the numerator (Smith 2019, pp. 164)."[3]

Smith's evaluation prompts two questions. Firstly, given his rejection of the mass of the attracting body, does Euler advocate a different law of gravity which lacks this variable? Secondly, if the answer to the first question is "yes," how might Euler deal with cases in which the distinction between the attracted and the attracting is less clear? One such case is the mutual attraction of Jupiter and Saturn. This study sets out to answer these question and clarify Euler's stance on the mass of the attracting

[1] Leonhard Euler to Tobias Mayer, 25 December 1751, in The Euler-Mayer Correspondence (1751-1755), ed. Eric G. Forbes, New York: American Elsevier, 1971, p. 44f.

[2] Another celebrated challenge to Newton's derivation of the law of gravity is from Roger Cotes, the editor of the second edition of the *Principia*, who points out to Newton that his application of the third law to gravity relies on a non-trivial assumption about the nature of gravity. For a helpful analysis of Cotes's query in relation to the measures of quantity of matter in the *Principia*, see Biener & Smeenk (2011).

[3] The same evaluation can be found in Harper (2011, pp. 367): "Euler, however, continued to object to Newton's inference to gravity proportional to the mass of an attracting body."

body and gravity. My findings run counter to Smith's evaluation. Euler does not reject the mass of the attracting body *per se* on my reading: instead, he rejects the way in which Newton derives it.

I substantiate my thesis as follows. In Section 1, I demonstrate that Euler shows no reservation in his scientific or philosophical works in acknowledging gravity's proportionality to the mass of the attracting body. In Section 2, I reproduce Euler's attempted reduction of gravity into a fluid force that behaves in accordance with Newton's law of gravity. I show too that scrutinizing Euler's reductionist program is especially relevant regarding Smith's ground-breaking findings on Newton's motivation for applying the third law to gravity.

1. Euler on the Mass of the Attracting Body: a Dismissal?

This section clarifies Euler's point of disagreement with Newton. Firstly, I suggest an alternative reading of Euler's "crude hypothesis" complaint in the 1751 letter. On My reading, Euler does not find the mass of the attracting body problematic *per se*. What he finds problematic is instead Newton's appeals to the third law of motion to derive this term. Secondly, I draw attention to a hitherto under-studied treatise by Euler on natural philosophy: *An Introduction to Natural Science*.[4] Euler theorizes in this treatise about the properties and modes of action of the matter that constitutes bodies and the surrounding ether. By doing so, he accounts for the three variables in Newton's law *quantitatively*. I provide relevant details of Euler's theory in Section 2.

Euler makes two points regarding the mass of the attracting body in the numerator in his letter to Mayer. There are different ways of interpreting what the two points jointly reveal about Euler's stance on gravity. The first point concerns the lack of empirical evidence for gravity's proportionality to the mass of the attracting body. In a system of non-contiguous bodies where the only force at work is gravity (such as our

[4]This treatise is written in German and only published posthumously. While it proves a rich collection of Euler's views on various issues in natural philosophy, the exact year of its composition remains unknown. Callinger (2015, pp. 402) suggests that this work can be completed "as early as 1755, but more likely a few years later." He also maintains that part of the reason why this work was not published is due to Euler's unwillingness to quarrel with a recently deceased, Christian Wolff, who died in 1754. Van Luntheren (1991, pp. 107) holds that the work is "probably composed during the years 1756-1758." For a historical account of the ether mechanisms that Euler has adopted in different stages of his career, van Luntheren (1991) is a comprehensive source.

solar system), there are no phenomena of motion that can show that gravity acts in proportion to the mass of the attracting body; nor can any terrestrial phenomenon be cited to support this proportionality.[5] Euler's second point is that Newton's method of determining gravity's proportionality to the mass of the attracting body presupposes a "crude hypothesis." Euler is targeting here Newton's application of the third law of motion to attraction in his derivation of the law of gravity.[6] This occurs in corollary 1 to Proposition 5 in Book III of the *Principia*:

> Proposition 5: The circumjovial planets [or satellites of Jupiter] gravitate toward Jupiter, the circumsaturnian planets [or satellites of Saturn] gravitate toward Saturn, and the circumsolar [or primary] planets gravitate toward the Sun, and by the force of their gravity they are always drawn back from rectilinear motions and kept in curvilinear orbits.
>
> Corollary 1: Therefore, there is gravity toward all planets universally. For no one doubts that Venus, Mercury, and the rest [of the planets, primary and secondary,] are bodies of the same kind as Jupiter and Saturn. *And since, by the third law of motion, every attraction is mutual*, Jupiter will gravitate toward its satellites, Saturn toward its satellites, the Earth will gravitate toward the Moon, and the Sun toward all the primary planets.[7] [my emphasis]

Smith does not dwell on Euler's letter. He uses it only to show that Euler is an ally of Stein in rejecting Newton's claim to have deduced the mass of the attracting body in the numerator from phenomena. Given Smith's conclusion, however, the following is a reasonable reconstruction of his assessment of Euler's attitude:

(1) Newton has no way of deriving the mass of the attracting body from phenomena.

(2) Newton instead hypothesizes that attraction is governed by the third law of

[5]This forms a stark contrast to the two other proportionalities in Newton's law. See Smith (2019) for a detailed discussion of the experiments Newton made to verify gravity's proportionality to the mass of the attracted body prior to the publication of the first edition of the *Prinicipia* in 1687. Gravity's inverse variation with distance squared is accepted after Alexis-Claude Clairaut's successful solution for the mean motion of lunar apogee in 1749, which Euler applauds as "the greatest discovery in the Theory of Astronomy." (Euler to Clairaut, June 1751)

[6]The literature on Newton's derivation of the law of gravity in the *Principia* abounds. See Stein (1990), Harper (2011, pp. 346-355), and Smith (2013). For Newton's initial argument for universal gravity in the manuscript *Liber secundus* – an earlier version of the Book III of the *Principia* composed in 1895, see Smith (2019) and Parker (2020). Parker (2020) is especially interesting for those wishing to learn about Newton's conceptual analysis of the third law of motion by analogy to magnetic attractions and the role this analysis plays in his attempt to infer the additivity of the active quantity of motion from that of the passive quantity of motion.

[7]See Newton (1999), p. 805. The literature on Newton's alleged deduction of this law from phenomena abounds. See for instance Duhem (1991), p. 190-195, Popper (1972), Stein (1990), and Harper (2011).

motion and elicits the mass of the attracting body from this hypothesis.

(3) Attraction being governed by the third law of motion is a crude hypothesis.

(4) We should therefore dismiss the mass of the attracting body from the numerator.

This is a perfectly natural reading of the given text. It can create difficulties in understanding Euler's publicly expressed views on gravity, however. The 1748 essay prize of the Royal Academy of Sciences, for instance, solicited from the participants a "theory of Saturn and Jupiter, whereby one can explicate the inequalities that these two planets appear to produce in each other principally during their conjunction."[8] Euler acknowledges in his submission to the prize that the inequalities in Saturn's motion are mainly due to its attraction toward Jupiter:

> [...] the inequalities in the motion of Saturn, one could conclude with good likelihood, that they are caused by the force whereby this planet is attracted to the body of Jupiter, which not only approaches Saturn the most, but also surpasses its quantity of matter, [...]. (Euler 1749 [1960], pp. 45)

Euler indicates in this quotation that the attractive force whereby Saturn gravitates toward Jupiter is dependent on two factors: firstly, the distance between Saturn and Jupiter; and secondly, Jupiter's quantity of matter. He then remarks that the inequalities in Jupiter's motion are similarly caused by "the force of Saturn over Jupiter." Note that in Euler's characterization, the force whereby one planet is attracted toward the other is dependent on the latter's quantity of matter as the *attracting* body. He further specifies the dependence relation in terms of proportionality:

> [...] to satisfy the proposed question, one will set out to determine the motions of three bodies that attract each other in the proportion of their masses and the inverse proportion of their distances squared, and put in the place of one of these three the Sun, and Saturn and Jupiter in place of the two others. (*ibid*, pp. 47)

As this quotation suggests, Euler holds that the attractive forces that celestial bodies exert on each other are proportional to their own masses: the mass of the attracting body, in other words. He later incorporates these proportionalities into his calculation, although the details of the procedure need not concern us here.[9] Euler's attitude toward

[8]All citations of Euler's submission to this prize comes from *Opera Omnia*, ser. secunda, XXV, pp. 45-157.

[9]For a helpful analysis of the history of Euler's engagement with the inequalities of Jupiter and Saturn's

the mass of the attracting body is also far from dismissive in his works of natural philosophy. In *Letters to a German Princess*, one of his most widely read texts, Euler states that "the power of attraction is proportional to the mass of the attracting body A, and to that of the attracted body B" in an alleged "demonstration" of Newton's law.[10] It is not impossible that Euler's 1751 letter to Mayer demonstrates a change of heart regarding whether the mass of the attracting body belongs in the numerator of the law of gravity. However, I think that it is more reasonable to seek an alternative reading of his complaint about the variable.

Fortunately, we have a valuable lead in the *Letters*. Euler suggests that while it is undeniable that some "(attractive) powers exist which are the causes of the reciprocal tendency of bodies toward each other," it remains controversial precisely *where* the attractive power resides:

> Thus, according to the first, the cause of the attraction resides in the bodies themselves, and is essential to their nature; and according to the last, it is out of the bodies, and in the fluid which surrounds them. (pp. 131)

The first group of philosophers comprises Newton and his followers; the second comprises Euler and like-minded advocates of "the principles of a rational philosophy" (*ibid*).[11] This short passage yields an important piece of information. Given Euler's use of the phrase "attractive power" that "resides in the bodies," it is likely that he is referring to Newton's considered views on attraction in *A Treatise of the System of the World* (1728) rather than those in the *Principia*. Consider the following passage from the *Treatise*:

> Since, however, the action of the centripetal force upon an attracted body be proportional, at equal distances, to the matter in this body, it is fitting to reason that it be

motion during conjunction with an emphasis on the mathematical innovations involved, see Wilson (1985), esp. Chapters 2-4.

[10]See (Euler 1840: Letter LVI). I put "demonstration" in scare quotes because it is not a demonstration in the sense of a derivation of each term in the law like the one Newton offers in Book III of the *Prinicipia*. It is instead a simple elucidation of how to understand the mathematical relations between those terms.

[11]Euler is here asserting his subscription to mechanical philosophy demanding causal explanation in terms of matter and motion only. In the eyes of his peers, Euler's commitment is perplexing and somewhat dogmatic. In a 1744 letter to Euler, Danille Bernoulli writes that "I cannot hide from you that on this point I am a complete Newtonian, and I marvel that you so long adhere to the Cartesian principles; perhaps some passion is mixed with it" (Fuss 1843, pp. 550-551). LaJoseph-Louis Lagrange and Jean D'Alembert, too, concur that Euler the great geometer presents himself as "a very bad philosopher" in the *Letters* in a 1769 exchange (*Ibid*, pp. 557-558). I obtained these valuable quotes from Wilson (2002, pp. 400-402).

proportional to the matter in the attracting body. Indeed, the action is mutual, and makes the bodies mutually approach one another by a mutual endeavor (*by the third law of motion*), and hence it must be the same *in* each body. [...]

And hence *the attractive force is found in both.* (Newton 1728, pp. 24, emphasis added)

Newton's claim that the action and the attractive force are to be found *in* the bodies us a celebrated interpretive challenge even today.[12] We find no such discussion about gravity's causal basis in the *Principia* and Newton himself suppressed the *Treatise* from publication: these facts also complicate the matter. From how Euler characterizes his point of disagreement, however, it is clear that he takes Newton to mean that the seat of attractive forces is literally present *in* the bodies in the sense that the bodies exercise attracting and that the attractive forces they exercise are always proportional to their quantity of matter. To Euler, this amounts to admitting that material bodies are capable of unmediated action across distance: a position with which he never made peace.[13] To identify the source of the attractive forces, according to Euler, we must instead look outside the bodies and turn to the surrounding medium, ether, which acts by contact on the bodies that are placed in it and makes them appear to attract each other.

A more defensible reading of Euler's 1751 letter to Mayer is as follows. Instead of dismissing the mass of the attracting body in the numerator *per se*, Euler is objecting to Newton's way of deriving this term. He objects to the application of the third law to attraction, as well as the concomitant position that the attractive forces reside in the bodies. As an ether theorist, then, Euler is presented with the difficulty of reconciling his acceptance of the law of gravity, on the one hand, and his rejection of Newton's way of deriving it, on the other.

[12]For some recent attempts to clarify the implications of these passages, see Janiak (2008), Schliesser (2011), and Kochiras (2013).

[13]I learn from [name and occasion omitted for anonymizing purpose] that Euler attributes the Moon's secular acceleration to the resistance from an aetherial fluid filling celestial space in the 1772 memoir, which is among the last works in his lifetime. For a helpful account of the various mechanical models Euler devises to explain phenomena involving action-at-a-distance (e.g., fire, magnetism, gravity, electricity, and the transmission of light), see Wilson (2002, pp. 400). As Curtis Wilson puts it neatly, Euler is quite aware that his dismissal of action-at-a-distance is already "an embattled view" during the 1740s but nonetheless retains his conviction throughout his life.

2. Euler's Alternative: Gravity as a Fluid Force

Although Euler is not Smith's (2019) primary figure of interest, one of Smith's significant discoveries can provide an interesting angle for assessing Euler's ether mechanism for gravity. According to (Smith 2019, pp. 176), close analysis of Newton's manuscripts reveals that "what lay[s] behind his resting so much weight on the third law was his conception of forces of nature as forces of interaction, and that what led him into this conception was the need to accommodate the aspect unique to gravitational forces, their seemingly proportioning themselves to the bodies on which they act". Why does gravity always adjust its magnitude according to the quantity of matter of the bodies on which it acts? On Smith's reading, Newton's answer to this question would be that although gravity acts on a body from without, it forms a correlative pair with the body's own exercised inherent force. Since such a pair of forces is governed by the third law, which means that gravity is equal in magnitude and opposite to the body's exercised inherent force, and the body's inherent force is proportional to its quantity of matter, it therefore makes sense that gravity always "proportions itself" [*se proportionne*][14] to the specific quantity of matter of the bodies on which it acts.

Newton is radical in how he accommodates gravity's proportioning itself to the quantity of matter of the bodies on which it acts. One way of appreciating this radicality, as Smith argues, is to recognize that this unique aspect of gravity is "a driving consideration of Einstein's theory of gravity in his theory of general relativity." Another way of appreciating Newton's radicality in this regard is to see how someone like Euler managed to accommodate the same feature. As we have seen, Euler rejected the application of the third law of motion to gravity and the conception of it as forming a correlative pair with the heavy body's inherent force.[15] How might Euler accommodate this unique aspect of gravity without appeals to the third law of motion?

Euler made several attempts throughout his career to reduce gravity to a fluid force.

[14]See Du Châtelet, "Exposition Abregée du Systême du Monde, et Explication des Principaux Phénomenes astronomiques tirée des Principes de M. Newton," pp. 47, appended to her translation, *Principes Mathématiques de la Philosophie Naturelle*, Par feue Madame la Marquise du Chastellet, 2 vol, Sceaux: Editions Jacques Gabay, 1990, a reprint of the 1759 edition.

[15]Euler's attitude toward the Newtonian conception of inherent force or *vis inertiae* has undergone a major change in the late 1740s. In his 1750 paper "Researches on the origin of forces," Euler argues against conceiving inertia as a force so as to reserve the term "force" for external causes capable of changing the state of a body. For a discussion of Euler's departure from the Newtonian conceptualization of inertia, see Stan (2017b) and Brading & Stan's "Body and Force," ms.

The most elaborate attempts occur in his unpublished treatise, *Introduction*. Seven chapters of the *Introduction* are devoted to theorizing the matter that constitutes bodies and the surrounding ether, respectively. He does so in order to recover the three variables in Newton's law of gravity quantitatively from the ether's material properties and its local action on bodies. This text also provides an ideal source to witness the tremendous challenge faced by an ether theorist attempting to accommodate gravity's proportionality to the mass of the attracted body. The challenge compels Euler to develop a highly complex theory of matter, the details of which can seem arbitrary at times. In what follows, I introduce the basic elements of Euler's theory that are necessary to understand how the ether acts on bodies, reproduce his attempt to recover the three variables on Newton's law based on that theory, and offer a critical evaluation of the whole attempt.[16]

Euler states in the opening of his chapter "On Gravity and the Forces acting on Heavenly Bodies" that he aims to explain gravity in terms of "unequal pressure of the ether, which increases with increasing distance from the earth." (§140) Given this aim, the main challenge he faces is the following: since it has been empirically established that gravity acts in proportion to the heavy bodies' quantity of matter, whereas fluid pressure does not, how can gravity be reduced to fluid pressure? Euler proposes a dualist theory of matter to meet this challenge. He also offers a different way from Newton's to accommodate gravity's proportioning its magnitude to the quantity of matter of the bodies on which it acts.

According to Euler, there exist only two types of matter underlying "the great variety of bodies" (§97) we find in experience: coarse matter and subtle matter. All bodies are porous, and the variety of bodies is due to the "quantity, size and arrangement of the pores that are distributed between the coarse parts" (§97) with the pores filled with subtle matter (§95). Importantly, coarse matter and subtle matter do not differ in size, shape, and motion, but in kind: they have distinct physical properties and also behave differently under impressed forces. Three distinctions demand special attention for my purposes here.

[16]For a closer analysis of Euler's theory of matter on its own right, see Gaukroger (1982), which focuses on Euler's account of impenetrability as the origin of impressed forces, and Brading & Stan's "Body and Force" ms., which reconstructs Euler's answer to "the problem of bodies," a research program demanding a causal explanation of collision in terms of the properties of material bodies and contact action.

The first distinction concerns the relationship between the types of matter and the bodies they constitute. While bodies are constituted by both coarse matter and subtle matter, according to Euler, only coarse matter "moves together with the body" it constitutes; subtle matter only "occupies a part of the body's extension," but does not contribute to the body's inertia[17] and quantity of of matter (§92). Therefore, a force acts on a body only insofar as the body contains coarse matter. That is why coarse matter can also be referred to as the body's "proper matter" and subtle matter as the "foreign matter" (§92).

The second distinction concerns density. Euler holds that the density of coarse matter is a constant, which has a value far greater than that of the density of gold (§96). Subtle matter by contrast not only "has a density many thousand times smaller" than coarse matter, but also admits of varying density (§96).

The third distinction concerns the matters' behaviors when acted on by an externally impressed force. Subtle matter is highly elastic, according to Euler, while coarse matter is devoid of elasticity. More importantly, the subttle matter in our universe always remains "in a forced state, and is compressed beyond its natural density" (§106). As a consequence, Euler posits, "subtle matter always exerts everywhere an unusually strong spring force and compresses all bodies" as we find it in nature (*ibid.*).[18] An immediate concern one might have at this juncture is that for the highly elastic subtle matter to remain compressed, the universe must be spatially bounded to prevent the ether, whose constituents are subtle matter, from expanding indefinitely. Euler is aware of this problem, but declines to provide an explanation for it: there seems to be no good way of getting around this difficulty.[19] Since the purpose here is to see how Euler uses his matter theory to deal with gravity, let us grant as a boundary condition of the theory that the ether filling the universe is and will remain compressed and compressible.

We are now in a good position to see what gravitation amounts to and how gravi-

[17]Euler's original terminology is *persistence*, which is defined as the property whereby all bodies "remain in their states" (§31). He criticizes the name "inertia" for carrying the "inappropriate" sense that only bodies at rest have it (*ibid.*).

[18]One thing left unanswered is how the change of density varies with the force impressed on subtle matter, a knowledge Euler admits not possessing (§102). Nor is it clear what would happen if subtle matter of different densities act on each other.

[19]See §106 for Euler's disclaimer that such questions "do not concern natural science" as they pertain to how "the divine work of creating and maintaining the world."

tation, which arises from the pressure of ether, behaves in accordance with Newton's law. Euler begins by inviting us to suppose that the ether is in a state of equilibrium when devoid of any body, with a certain pressure h, such that a test particle placed in it "would be pressed equally from all directions, and would thus not be set in motion" (§140). Suppose also that we placed two bodies in it. One of these bodies is a mid-size object and the other is considerably more massive, such as the Earth. To make the former move toward the latter in accordance with Newton's law, Euler makes two more assumptions:

(1) The coarse matter the Earth contains causes a decrease of pressure in the ether (§145).
(2) The decrease is proportional to the Earth's quantity of (coarse) matter (§145) and inversely proportional to the distance between the Earth and the object (§140).

With these assumptions, we then know that given the initial pressure h the pressure at any distance x from the Earth would be $h - \frac{A}{x}$, in which A is a constant. One might ask *why* and *how* the coarse matter in the Earth disturbs the ethereal equilibrium to create varying pressure that satisfies these relationships. Euler unfortunately deferred this curiosity to a fuller matter theory awaiting future research (§146). For this study, let us take the assumptions as true and see, with Euler, how to calculate the gravitational force acting on the object.

Suppose that the coarse matter the object contains can be piled up to make a cube of the size $a^2 b$, in which b denotes the height of the cube and a^2 the two surfaces orthogonal to the line connecting the center of the Earth and that of the cube. In calculation, the net fluid force acting on this imaginary cube, denoted as P, would give us the gravity of the object in question, which is simply the difference of forces acting on the cube's two surfaces:

$$P = a^2(h - \frac{A}{x+b}) - a^2(h - \frac{A}{x}) = a^2(\frac{A}{x} - \frac{A}{x+b}) = \frac{a^2 b A}{x(x+b)}$$

If we further suppose that a^2b is numerically equal to c^3, we get:

$$P = \frac{Ac^3}{x(x+b)}$$

Since, as Euler holds, b is so small in compared to x (§142), we can rewrite the equation as:

$$P = \frac{Ac^3}{x^2}$$

How is this expression mathematically equivalent to Newton's law of gravity, which states that gravitational forces are proportional to the mass of the attracting body, the mass of attracted body, and inversely proportional to the distance squared? First, the distance squared already appears in the denominator, so it needs no further attention. Second, Euler holds with respect to the mass of the attracting body that A "will for the case of the Earth have a particular constant value" and stands "in the ratio of the [Earth]'s mass." (§145) Finally, regarding the mass of the attracted body, recall that Euler's theory of matter posits that the density of coarse matter is a constant and that bodies' proper quantity of matter is the quantity of coarse matter constituting them. Therefore, c^3, which is the volume of the coarse matter in the attracted body, is in a fixed proportion to the body's quantity of matter. The challenge to accommodate gravity's always adjusting its magnitude to the bodies on which it acts is therefore addressed, and all three variables in Newton's law of gravity are recovered without appeals to the third law of motion.

The Eulerian conception of attraction or gravity is obviously very different from its Newtonian counterpart. Take the Sun and Jupiter for example. On Newton's characterization in the *Treatise*, it is "one action by which the Sun and Jupiter mutually endeavour to approach each the other," and that action is governed by the third law of motion. Newton goes on to analogize this action to the contraction of a cord stretched between two bodies, thus making concrete what it is for two bodies to engage in one action. For Euler, by contrast, the mutual attraction is due to the ether's action on the Sun and Jupiter respectively. It is in virtue of the coarse matter constituting the Sun and that constituting Jupiter that the ether pressure in their intermediary region becomes lower than that in other regions in space, so the two bodies are propelled toward each other as if they are attracting each other. The Eulerian gravity is universal in a distinctive sense: as the coarse matter constituting each celestial body causes the ether pressure in its "sphere of influence" to decrease, bodies always gravitate toward regions of space that have lower pressure than the ones they themselves occupy.

Euler offers us a very different and certainly interesting way to conceptualize gravity. His reductionist program is far from satisfying, however. First and foremost, what we find in the *Introduction* is hardly an alternative derivation to Newton's of the law of gravity, but merely a reverse engineering. Taking the three variables in Newton's law as given, Euler provides a way of developing a theory of matter that is adequate to account for each of them. He posits numerous assumptions while doing so to satisfy the desired mathematical relations. Consider the mass of the attracting body again. Instead of hypothesizing the the third law holds for attraction to derive this term as Newton does, Euler posits that the coarse matter constituting celestial bodies causes the ether pressure to decrease in a very specific way. As for why coarse matter can cause the ether pressure to decrease, however, he admits agnosticism (§146). Note moreover that in his expression for gravity, $\frac{Ac^3}{x^2}$, none of the terms stand for an empirically quantity apart from x, the distance. The most we know about A and c^3 is that they are in a fixed proportion with the mass of the Earth and the mass of the heavy object respectively, but these relationships are postulated, not empirically proven. Finally, although its mathematical agreement with Newton's law is impressive, Euler's expression obtains only for cases in which one body is massive enough to cause the ether pressure to decrease and the other is not. For two massive bodies such as Jupiter and Saturn, Euler would probably declare that his theory still falls short of recovering the mathematical relations involved. These limitations cannot escape him.

One might think that despite these limitations, Euler's ether hypothesis was not altogether a mistake given the state of knowledge at the time. Like the Newtonian hypothesis that gravity is a force of interaction governed by the third law, it was not impossible that future research could vindicate Euler's ether theory, according to this stance. At most, we ought to say that the interaction hypothesis and the ether hypothesis were two competing theories of gravity that both lacked empirical support but were perfectly consistent with the state of knowledge in the 1750s.

I do not believe that this position is fair to Newton for two reasons. Firstly, Smith (2019) invokes compelling evidence to argue that, while Newton did not have direct evidence showing that gravity is a force of direct interaction governed by the third law, he did obtain impressive evidence demonstrating that the third law holds for impact by conducting elaborate experiments on a range of non-perfectly elastic spheres. Thus, Newton gave an added reason to count on the third law's holding universally – including for gravity. By contrast, the existence of ether is founded on the conviction that explaining gravity requires it, and the properties of ether, as has been shown, are postulated to meet the demand of recovering the three variables in the law of gravity. Therefore, while Euler is right to point out the lack of *direct* empirical support for planetary interaction, from an evidentiary point of view, his ether hypothesis nevertheless falls short of its Newtonian rival.

Second, and more importantly, while it may be true that the two hypotheses were consistent with the state of knowledge in the 1750s, even a cursory comparison of the research programs they respectively call for suffices to reveal that the two hypotheses cannot be safely regarded as competitors within one arena. On the one hand, Newton outlines an experimental in Proposition 92 of the *Principia* to test gravity's proportionality to the mass of the attracting body, and lays the ground for a research program leading to the identification of true motions through a series of successive approximation.[20] On the other hand, Euler's ether hypothesis yields no such research programs to test its own viability, nor does it offer helpful resources to mobilize quantitative data about the physical world to solve research questions. Of course, the lack of fruitfulness did not escape Euler. In this concluding remark in the chapter on gravity, Euler admits that, since the ether hypothesis imposes question about coarse matter (which are unlikely to be answered), its virtue lies elsewhere than fruitfulness to future research:

[20]See Smith (2019, pp. 176) for the view that Proposition 92 lays out the program for testing this proportionality. The literature on Newton's scientific methodology abounds. For the method of approximation, which lies at the heart of Newton's methodology, see Smith (2014, pp. 262-345). Also see Harper (2011) and Smith (2011).

> Although we have to stop here and *can hardly hope ever to find the cause of the diminution of the elastic force of the ether*, it is easier to resign to this than to merely maintain that all bodies are by their nature endowed with a force to attract each other. (§146, my emphasis)

By the by, this realization may explain why Euler did not attempt another ether hypothesis of gravity of the same scale and sophistication as the one we have seen in the *Introduction* later in his career, although his conviction regarding its ultimate triumph remains.

Conclusion

In this paper, I have made both negative and positive use of Smith's "Newton's numerator in 1685: A year of gestation" from the perspective of Euler scholarship. First, I addressed the flawed conclusion that Smith draws regarding Euler's stance on the mass of the attracting body in the 1750s and thereafter. My study demonstrates that Euler's stance differs from – and is importantly subtler than – Smith's suggestion. Rather than rejecting the term's belonging in the numerator of Newton's law of gravity, Euler is instead concerned with Newton's derivation thereof, i.e., by applying the third law of motion to gravity and thereby hypothesizing that gravity is a force of interaction governed by the third law. Having clarified Euler's stance on gravity's proportionality to the mass of the attracting body, I then moved onto make use of an interesting finding in Smith's paper concerning Newton's motivation for applying the third law to gravity. On Smith's account, this maneuver constitutes Newton's radical way of accommodating gravity's adjusting its magnitude to the mass of the bodies on which it acts. From the perspective of Euler scholarship, I argue that the radicality could be better appreciated by recognizing how Euler accommodates the same feature of gravity differently, i.e., without appeal to the third law but to a highly speculative theory of matter. I will leave it to readers to decide whether Euler is justified in criticizing Newton's derivation of the law of gravity as "founded on a crude hypothesis". If the above analysis is on the right track, however, then we ought to declare Euler's alternative to be the cruder party by comparison with its Newtonian counterpart.

References

[1] Zvi Biener and Chris Smeenk. Cotes' queries: Newton's empiricism and conceptions of matter. *Interpreting Newton*, pages 105–137, 2012.

[2] Ronald S Calinger. *Leonhard Euler: Mathematical Genius in the Enlightenment*. Princeton University Press, 2019.

[3] Pierre Maurice Marie Duhem. *The aim and structure of physical theory*, volume 13. Princeton University Press, 1991.

[4] Leonhard Euler. *Letters of Euler to a German Princess, on Differnet Subjects in Physics and Philosophy*, volume 1. Harper, 1840.

[5] Leonhard Euler. Anleitung zur naturlehre. 1862.

[6] Leonhard Euler. *The Euler-Mayer Correspondence (1751–1755): A New Perspective On Eighteenth-Century Advances in the Lunar Theory*. Macmillan International Higher Education, 1971.

[7] Stephen Gaukroger. The metaphysics of impenetrability: Euler's conception of force. *The British Journal for the History of Science*, 15(2):132–154, 1982.

[8] William L Harper. *Isaac Newton's scientific method: turning data into evidence about gravity and cosmology*. Oxford University Press, 2011.

[9] Andrew Janiak. *Newton as philosopher*. Cambridge University Press, 2008.

[10] Hylarie Kochiras. Causal language and the structure of force in newton's system of the world. *HOPOS: The Journal of the International Society for the History of Philosophy of Science*, 3(2):210–235, 2013.

[11] Isaac Newton. *De mundi systemate liber*. J. Tonson, 1728.

[12] Isaac Newton. *The Principia: mathematical principles of natural philosophy*. Univ of California Press, 1999.

[13] Adwait A Parker. Newton on active and passive quantities of matter. *Studies in History and Philosophy of Science Part A*, 84:1–11, 2020.

[14] Karl R Popper. *Objective knowledge*, volume 360. Oxford University Press Oxford, 1972.

[15] Eric Schliesser. Without god: gravity as a relational quality of matter in newton's treatise. In *Vanishing matter and the laws of motion: Descartes and beyond*, volume 13, pages 80–102. Routledge, 2011.

[16] George E Smith. On newton's method: Book symposium on harper: Isaac newton's scientific method. *Metascience*, 22(2):215–246, 2013.

[17] George E Smith. Closing the loop. In *Newton and empiricism*, pages 262–352. Oxford

University Press, USA, 2014.

[18] George E Smith. Newton's numerator in 1685: A year of gestation. *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics*, 68:163–177, 2019.

[19] Marius Stan. Euler, newton, and foundations for mechanics. In Chris Smeenk & Eric Schliesser, editor, *The Oxford Handbook of Newton*, pages 1–22. Oxford University Press, 2017.

[20] Howard Stein. "from the phenomena of motions to the forces of nature": Hypothesis or deduction? In *PSA: Proceedings of the biennial meeting of the philosophy of science association*, volume 1990, pages 209–222. Philosophy of Science Association, 1990.

[21] Frans Herbert Van Lunteren. Framing hypotheses: Conceptions of gravity in the 18th and 19th centuries (phd thesis). 1991.

[22] Curtis Wilson. Euler on action-at-a-distance and fundamental equations in continuum mechanics. In *The investigation of difficult things: Essays on Newton and the history of exact sciences in honour of DT Whiteside*, pages 399–420. Cambridge University Press, 2002.